Design of hybrid computer for modelling of dynamic systems

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Abstract—This paper deals with the design of an analog computer with digitally controlled components for the simulation of linear and nonlinear dynamic systems or for the study of transfer functions of electronic filter or oscillator topologies. The device should allow partial adjustment using a standard computer and also record the measured results in digital form.

Keywords— analog computer, hybrid computer, PWL function, integrator, dynamic system

1. INTRODUCTION

The analog computer is the type of computer which operates with the continuous quantity, most often with voltage. It is based on the analogy between the mathematical description (most often in the form of differential equations) of a physical phenomena and the mathematical description of an electronic circuit [1]. The great advantage of analog computers is their simple implementation of complex problems that can be solved in real time. The solution obtained from an analog computer is not burdened with numerical error and the only problem remains the inherent noise of used components which is very small due to modern technology. The disadvantage of these analog systems is their absence of a method of storing the measured results and their eventual processing. This problem can be solved by a hybrid computer that combines analog parts with digital technology. For example, the measured results in analog form are digitized using an AD converter and these results are processed by available digital methods. However, the price for this method is the numerical error.

Analog and hybrid computers are devices with a long history and at first glance it may seem that this technology is becoming less important with the rise of digital technology. However, research and development departments around the world continue to use these devices. For example, paper [2] deals with the use of analog computer to study the processes between molecules. In recent years, new applications of analog computers in the optical domain have also emerged. Optical analog computer can be used for construction of neural network [3]. Similarly, in paper [4] nano-optical elements are used, whose properties enable them to solve the wave equations. This means that analog computers and analog technology in general have not had their last word.

2. BLOCK STRUCTURE

The hybrid computer implementation will contain separate blocks that will be connected by pins and wires. This solution allows a great versatility in which both a simulation of a 4th order differential equation and a set of four differential equations can be solved. The supply voltage is chosen to be symmetrical ±15 V for the analog part and 5 V or 3.3 V for the digital part, which allows operation in a large state space. Some blocks are designed as purely analog, others have analog functions supplemented by digital parts that allow either the setting of individual blocks or the sensing of analog quantities for digital processing. In the following sections, the individual components are commented in more details.

2.1. Integrators and summators

Four integrators represent the core of the whole hybrid computer due to the realization of continuous dynamical systems whose basic description are differential equations. Each of the integrators will be built from a commercially available AD844 element. When using the compensation clamp of this element, it behaves as a second generation positive current inverter (CCII+) with a decoupling voltage follower. The advantage of using this element is its easy implementation as an integrator with both positive and negative sign and broadband 20 MHz bandwith. The change of the sign will be realized by a switch. The switches will also be used to select the desired time constant from four fixed implemented and one selectable which will be accessible by pins. This solution allows using the custom arbitrary value of the time constant or plug a fractal element [5] into the integration component, which increases the versatility of the computer. The integrators will also include an initial condition setting block (IC block), which will be only one and will be implemented as a switchable block. The whole integrator can be described by the equation

$$u_{out}(t) = \pm\frac{1}{\tau}\int u_{in}(t)dt + u_{IC}(0) \tag{1}$$

where τ is a time constant of integrator and u_{IC} is a initial condition.

The summators will be realized fully analogue and will be a standard summing amplifier with 6 inputs. The output of the amplifier can be routed through an inverting amplifier so that both positive and negative variants can be implemented. When creating a summing amplifier from an operational amplifier, the equation for the sum voltage can be written as

$$u_{out}(t) = \pm\sum_{n=1}^{N} u_n(t) \tag{2}$$

where $N = 6$ (number of inputs) and n is a index of input.

2.2. Multipliers and potentiometers

The proposed analog computer should mainly be used for simulations of nonlinear dynamic systems with chaotic or hyperchaotic behavior. The nonlinearities used in these systems are mostly in a polynomial form, which can be very easily implemented using standard available analog multipliers. For this application, the well-known IC AD633 was selected due to voltage range ±15 V which covers the entire state space. This component solved the equation

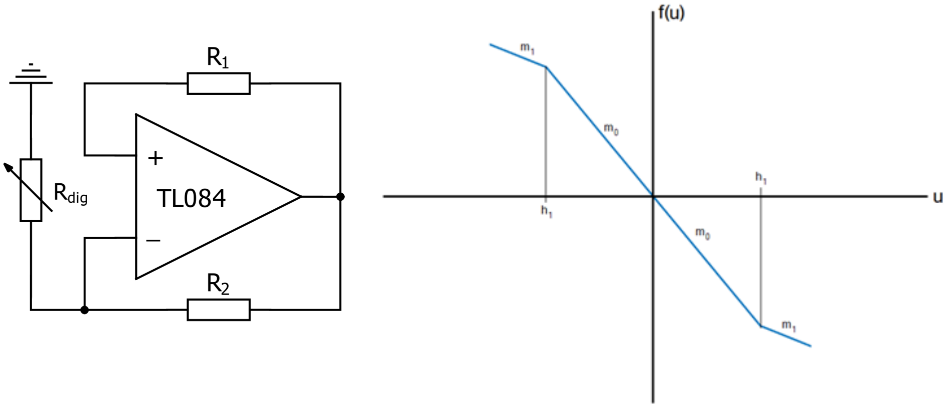
$$W = \frac{(X1 - X2)(Y1 - Y2)}{K} + Z \tag{3}$$

where X, Y, Z are input voltages (X, Y are realized as differential inputs) and $K = 10$ is the internal division constant. This constant is must be taken into account when programming the computer and preparing the programming scheme.

Potentiometers will be implemented as voltage dividers. This configuration allows to generate dimensionless parameter values independent of the current and voltage ratios of the circuit. The range of parameters will be allowed to be 0-1, so the parameters of dynamic systems will have to be normalized to 1. In addition to standard potentiometers, two digital potentiometers will be integrated in the voltage divider configuration, which will be controllable both via internal controls and via application in PC.

2.3. PWL block

As mentioned above, it is necessary to create nonlinearity on an analog computer when the nonlinear dynamic systems be simulated. A frequently used type of nonlinearity is the so-called *piecewise linear function* (PWL function). As can be seen in the fig. 1b, there are linear sections with a given slope that changes at certain points. Commonly, the PWL function is formed by a circuit with operational amplifiers, but these configurations have a major disadvantage in the number of points where the slopes change because each point needs at least one operational amplifier. The planned PWL block would advantageously use a digital potentiometer in negative impedance converter circuit and AD converter. Using the control circuit, the resistor value is set to a value that corresponds to the desired slope. AD converters would monitor the voltage value across the PWL block and when the point set by the control circuit is exceeded, the digital potentiometer value changes to the new slope. The PWL block created in this way has the advantage not only of using only one operational amplifier, but also of being able to construct a large number of points and sections with a wide variation of slopes. With a sufficiently small step between points, functions that are not directly labeled as PWL can be approximated very well (e.g. realizing the $\sin x$ function approximated by the PWL function with a fine step).



(a) (b)

Figure 1: Simplified schematic of PWL block and example of PWL function

2.4. IC block and digital processing

The IC block will provide the initial conditions settings. This block will contain digitally controlable voltage source that can be realized by digital potentiometer in voltage divider configuration and operational amplifier follower. This structure will precharge the capacitors occurring in integrators to the required voltage level. It is not necessary to construct the IC block for each integrator separately, but it is possible to set all integrators to the available capacitor pin mode using a switch and gradually precharge all capacitors. The IC block will control by digital processing part.

The block labeled as *digital processing* will be the main controlling element of the hybrid computer. The main tasks of the control logic include setting parameters for the PWL block, operating AD converters to digitize the signal from the output nodes, controlling the display, operating the IC block to set initial conditions, controlling parametric digital potentiometers and communicating with a standard desktop PC application. For this purpose, a circuit from the STM32 series will be selected, whose characteristics and quality support from the manufacturer fully suit the application. Manufacturer support offers programming of the device using three different methods: low-level using CMSIS, low-layer (LL) and high-level High Application Layer (HAL). The use of HAL in combination with the STM CubeMX environment greatly simplifies the development of the entire program, because it is not necessary to directly set up the registers to set up the various peripherals, but this system will set up the registers automatically by using predefined functions. This way of programming saves a lot of time and allows you to concentrate more on optimizing the desired program.

The computer mode will be divided into two basic modes: offline mode, where all control of the computer takes place directly on the computer and data is displayed on the screen, and online mode, where the digital part is controlled from the desktop application. The online mode will be functionally identical to the offline mode, but will be complemented by the ability to upload measured results directly to the app. The measurement results will be written to a file (probably a CSV file), which will allow their subsequent processing, for example using Matlab.

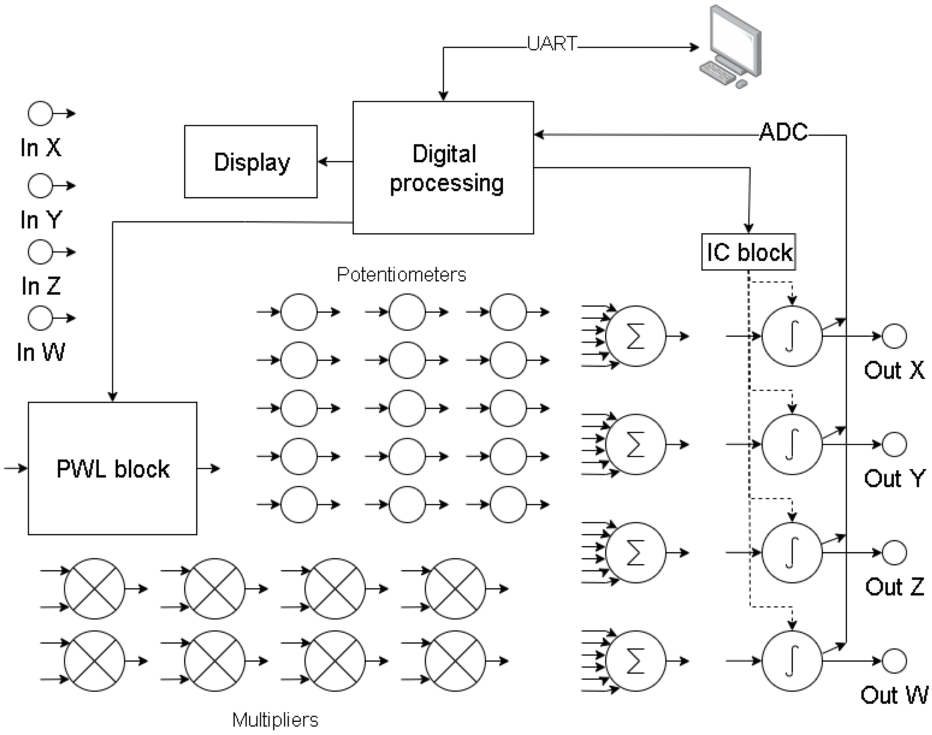


Figure 2: Block diagram of the designed hybrid computer

3. EXAMPLE OF SIMULATION

As an example of working with an analog computer, it is possible to show the solution of a simple ordinary second order differential equation which describe a behavior of electric harmonic oscillator

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{4}$$

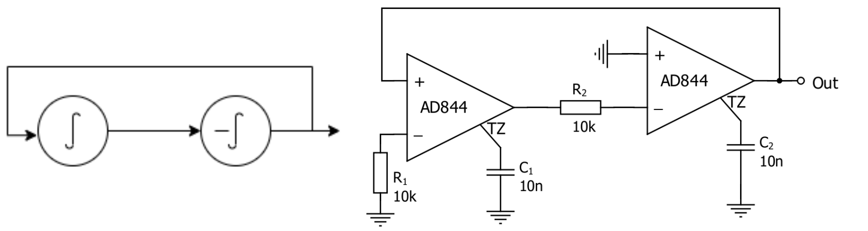
where x is a state variable which can be replaced by voltage and ω is a constant representing the oscillation frequency value. It can be shown [6] that ω can be calculated as

$$\omega = \sqrt{\frac{1}{\tau_1 \tau_2}} = \frac{1}{R_1 C_1 R_2 C_2} \tag{5}$$

where τ_1, τ_2 are time constants, in this case $\tau = RC$ with elements of integrators. It is possible to achieve harmonic oscillations by introducing positive feedback. The form of the solution of the differential equation under positive feedback can be written as

$$x = e^{\alpha t} \sin \omega t \tag{6}$$

Fig.3b shows a possible implementation using two integrators consisting of AD844 circuits connected as current conveyors. The circuit in the left part implements a positive integrator with time constant $\tau = RC = 10^4 \cdot 10^{-8} = 100\ \mu$s. It is followed by an inverting integrator with the same time constant. After the integrator, the output signal of periodic oscillations is already taken. This circuit was simulated in PSpice and result of the simulation is shown in fig. 4. Oscillation frequency $f \approx 1.58$ kHz coresponds with used time constant. The slight deviation from the exact frequency value of 1.5915 kHz is due to the effect of real element used, which affects the value of time constant by its parasitic capacitances. In future use of these elements, the resistors determining the time constant will be replaced by trimmers to ensure accurate setting of the desired time constant. Note that the oscillations do not have a constant amplitude, which corresponds to the case where alpha is negative. This fact is due to the use of realistic marcomodels of the integrated circuits, which include parasitic properties and decrease the energy from the dynamic system. This effect could be compensated by adding an amplifier with automatic amplitude stabilization, however, it is not necessary to implement an amplifier to see and understand the problem.



(a) (b)

Figure 3: Realization of differential equation: a) programming scheme, b) circuit implementation

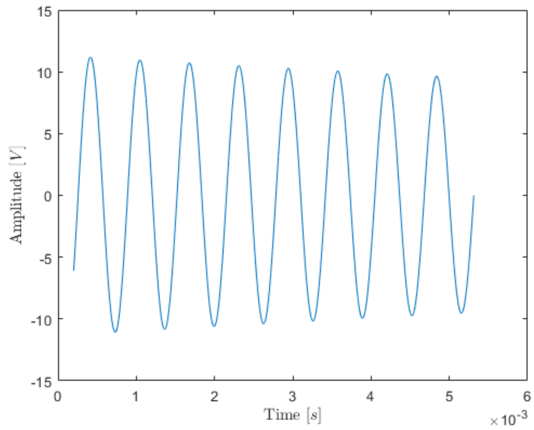


Figure 4: Output voltage waveform (solution of differential equation) simulated in PSpice

4. CONCLUSION

This paper briefly describes the design of hybrid computer which can be used for modelling of linear or nonlinear dynamic system. The presented computer should cover a large number of applications. Thanks to the implementation of a digitally controlled PWL block with a fine step, even functions of a more complex character that would normally be difficult to implement can be approximated very accurately. A great advantage will also be the ability to set initial conditions from the digital part of the circuit and the ability to record results using AD converters. A simple example of a harmonic oscillator was used to show how dynamic systems can be implemented in electronic circuits and used to find solutions.

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